# SC 2014 Doctoral Showcase

Akhil Langer (alanger@illinois.edu)

## Parallel Algorithms for Two-stage Stochastic Integer Optimizations

Akhil Langer[†], Udatta Palekar[‡](Co-advisor) and Laxmikant Kale[†](Advisor)

[†]Department of Computer Science, [‡]Department of Business Administration

University of Illinois at Urbana-Champaign

{alanger, kale, palekar}@illinois.edu

**Introduction** In many real world situations, future outcomes such as weather in agriculture, product demands in the manufacturing industry, stock prices for an investor, etc. are dependent on myriad different factors that cannot be deterministically predicted. However, resource allocation has to take place before the actual realization of these unknown parameters. When resource use has to be optimized under such conditions, the resulting problem is called stochastic optimization. It is called as stochastic integer optimization when the resources are to be allocated in integral units. Unlike deterministic programming, stochastic programming explicitly incorporates uncertain parameters by assuming a probabilistic distribution to make a more rational decision in problems that require, for example, an optimal allocation of resources. Applications of stochastic programming spans a diverse fields ranging from production, financial modeling, transportation (road as well as air), supply chain and scheduling to environmental and pollution control, telecommunications and electricity.

**Approach and Challenges** To model the application with fidelity requires hedging against hundreds to several thousands or more scenarios, which gives rise to exceptionally computationally challenging problems. The novelty of our work is in combining high performance computing and stochastic optimization to solve large-sized stochastic integer optimization problems that have not been tackled before. We use branch-and-bound which is a standard approach for solving integer programs. We propose a nested parallelism approach that exploits the inherent scenario parallelism in stochastic optimization along with the parallel exploration of branch-and-bound tree vertices. This is unlike traditional branch-and-bound applications (such as integer programming, traveling salesman problem, game tree search, etc.) in which each tree vertex is an atomic unit of work. In our approach, each vertex requires solving several iterations of Bender's decomposition. Each iteration is comprised of two stages that are mutually exclusive and are executed over different processes. A Stage 1 process saves the state of the vertex (for later retrieval) that is currently in its Stage 2 and switches to another vertex. This interleaved evaluation of the tree vertices is required in order to ensure high utilization of compute resources.

Stochastic integer optimization programs have unique characteristics that are different from the challenges faced in the typical, iterative, scientific applications. These characteristics make it very difficult to realize a scalable design. Some such characteristics are coarse and unpredictable grain sizes, large memory footprints, variable amount of available computation, and easily perturbable branch-and-bound trees. Network noise and operating system interference can affect the order in which messages are sent and delivered to processes. This alters the order in which vertices are explored and hence the solution of the degenerate linear programs as the linear program solvers use advanced start which starts from the solution of the previous linear program optimization. This makes it difficult to tune performance as we get different branch-and-bound trees across multiple trials of identical configuration. Additionally, in this application, better utilization of processors does not necessarily mean faster times to optimal solution.

**Results** US Military has to allocate its 1300 aircraft to different wings and missions. These allocations are to be done while accounting for humanitarian/wartime contingencies, cargo fluctuations, aircraft breakdowns, and short-notice special missions. We modeled this problem as two-stage stochastic integer program that gave us a cost benefit of up to 6% (equivalent to several hundred million dollars) over deterministic optimization. It is important that these problems be solved quickly while evaluating large number of scenarios. Our attempts results in strong scaling to hundreds of cores for these datasets. The application achieves speedup of up to 32x while scaling from 3 to 480 cores and has a parallel efficiency of greater than 0.4 at 120 cores in 100% of the trials. In contrast, commercial state-of-the-art Integer Program solver called Gurobi is shown to achieve only 10% parallel efficiency on 32 cores.