Comparing algorithms for detecting abrupt change points in data

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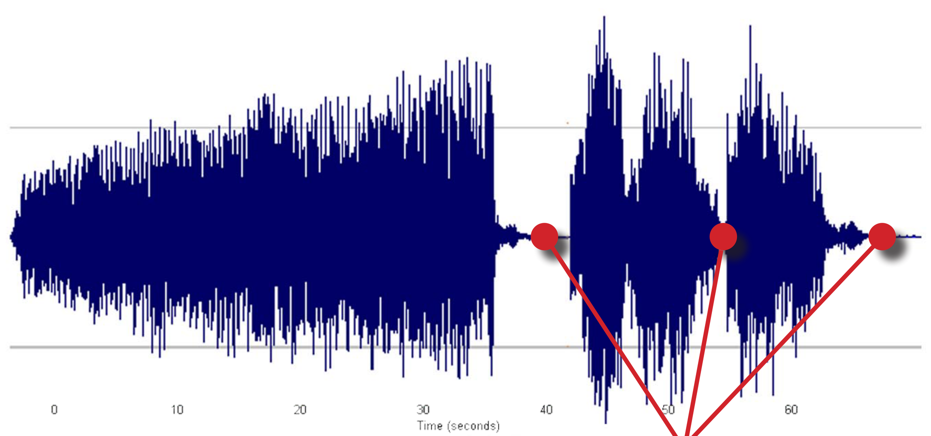
1/7 Introduction

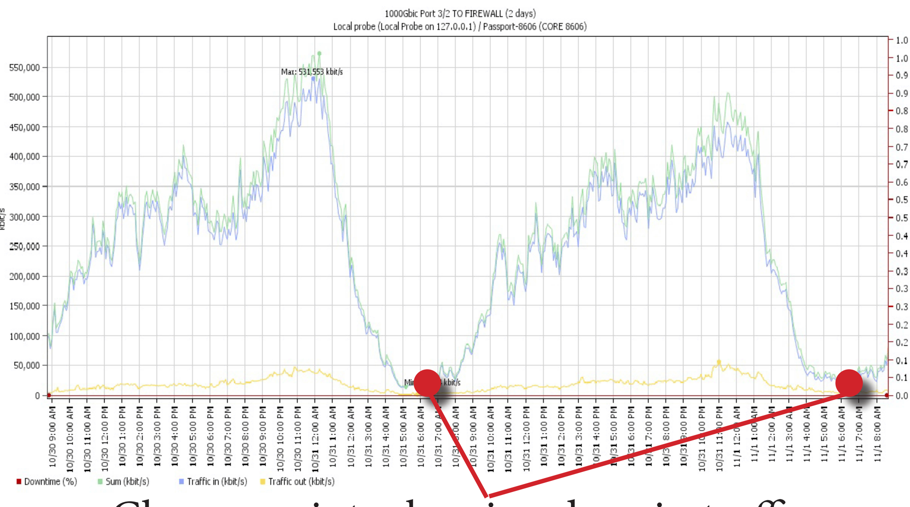
Many data-centric applications produce time-stamped streams of data ripe for analysis, and a key aspect of this data is understanding when the underlying distribution producing this data changes. These moments of change are called “change points” and have a variety of uses from fault detection to enhanced forecasting to classification and many others.

Musical Note Segmentation



Change points between notes

Denial-of-Service Detection



Change points showing drop in traffic

2/7 The Algorithms

Galeano and Peña’s Likelihood Ratio Test [1]

Fit a VARMA model to the data, and extract the residuals $\boldsymbol{e_i}$ from the data. For some point in time $\boldsymbol{h}$, calculate the LRT test statistic, and compare against the critical value for that dimensionality.

$$LRT(h) = n \ln \frac{\left|\frac{1}{n}\sum_{i=1}^{n} e_i e_i'\right|}{\left|\frac{1}{h}\sum_{i=1}^{h} e_i e_i'\right|^{\frac{h}{n}} \left|\frac{1}{n-h}\sum_{i=h+1}^{n} e_i e_i'\right|^{1-\frac{h}{n}}}$$

Galeano and Peña’s CUSUM Test [1]

Same as LRT but with the CUSUM test statistic.

$$C_\ell^r(h) = \frac{h}{\sqrt{2k(r-\ell+1)}} \left(\frac{\sum_{t=\ell}^{h} e_t (\hat{\Sigma}_\ell^r)^{-1} e_t'}{h} - \frac{\sum_{t=\ell}^{r-\ell+1} e_t (\hat{\Sigma}_\ell^r)^{-1} e_t'}{r-\ell+1} \right)$$

Desobry et al.’s Kernel Change Detection [2]

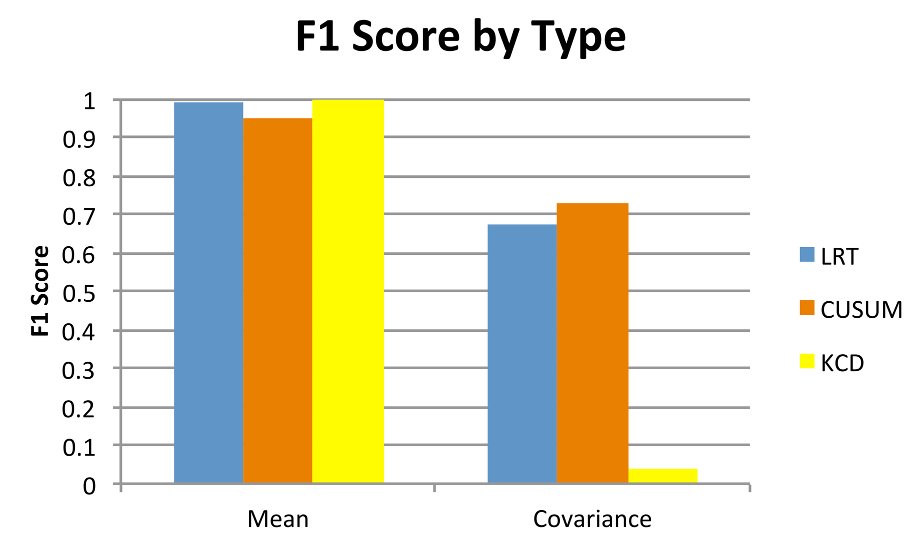
Given a data window of size $\boldsymbol{2m}$, fit two one-class SVMs to the first $\boldsymbol{m}$ points and second $\boldsymbol{m}$ points, and use the KCD statistic to calculate dissimilarity between the two data sets.

$$KCD(h) = \frac{\arccos\left(\frac{\boldsymbol{\alpha}_p^T K_{p,f} \boldsymbol{\alpha}_f}{\sqrt{\boldsymbol{\alpha}_p^T K_{p,p} \boldsymbol{\alpha}_p}\sqrt{\boldsymbol{\alpha}_f^T K_{f,f} \boldsymbol{\alpha}_f}}\right)}{\arccos\left(\frac{\rho_p}{\sqrt{\boldsymbol{\alpha}_p^T K_{p,p} \boldsymbol{\alpha}_p}}\right) + \arccos\left(\frac{\rho_f}{\sqrt{\boldsymbol{\alpha}_f^T K_{f,f} \boldsymbol{\alpha}_f}}\right)}$$

3/7 Changes in Mean vs. Covariance

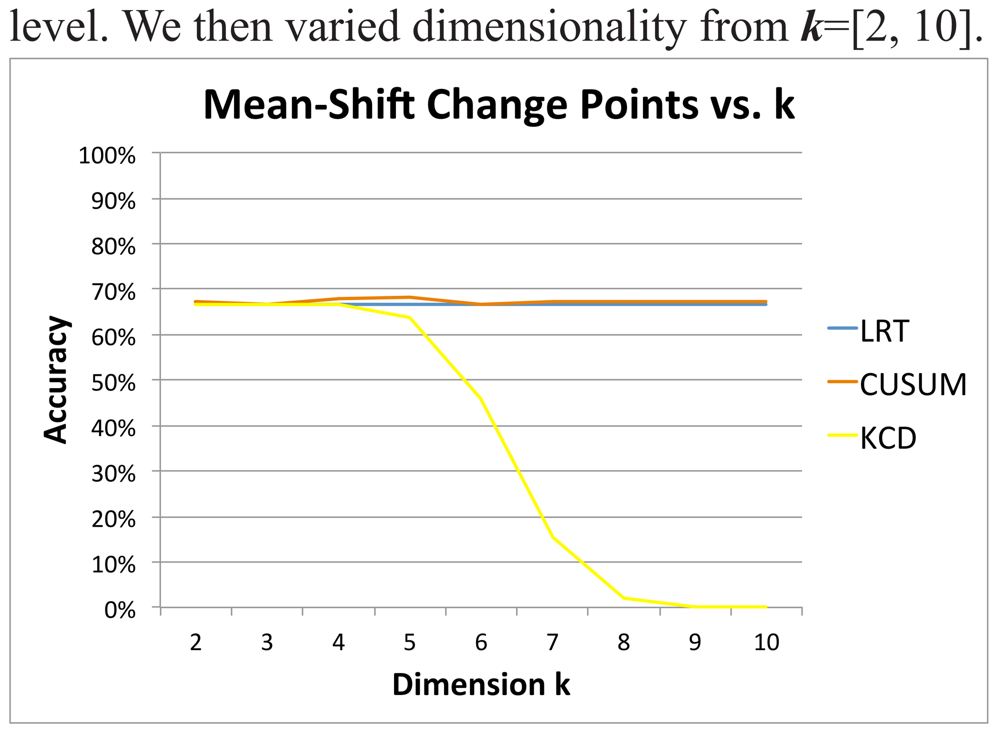
For covariance changes, we generated two regimes of data with constant mean and different covariance matrices. KCD then fit one-class SVMs to the covariance matrices within the past and future windows. Mean shifts rather used random means and constant covariance.

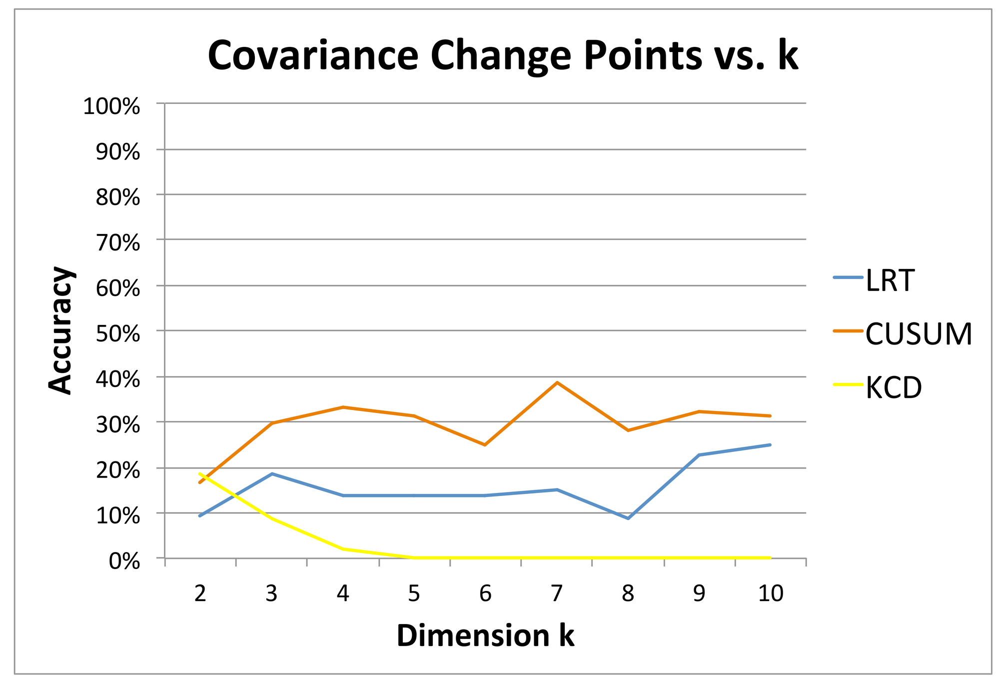
We simulated 500 bi-variate data points with a change point at $\boldsymbol{h}$=250, KCD window size of 400 ($\boldsymbol{m}$=200), and compared the LRT and CUSUM test statistics at the 95% confidence level.



4/7 Sensitivity to Dimensionality

Once again, we simulated 500 multi-variate data points but included change points at $\boldsymbol{h}$={125, 250, 375}. We left the KCD window size at 400 ($\boldsymbol{m}$=200), and compared the LRT and CUSUM test statistics at the 95% confidence level. We then varied dimensionality from $\boldsymbol{k}$=[2, 10].

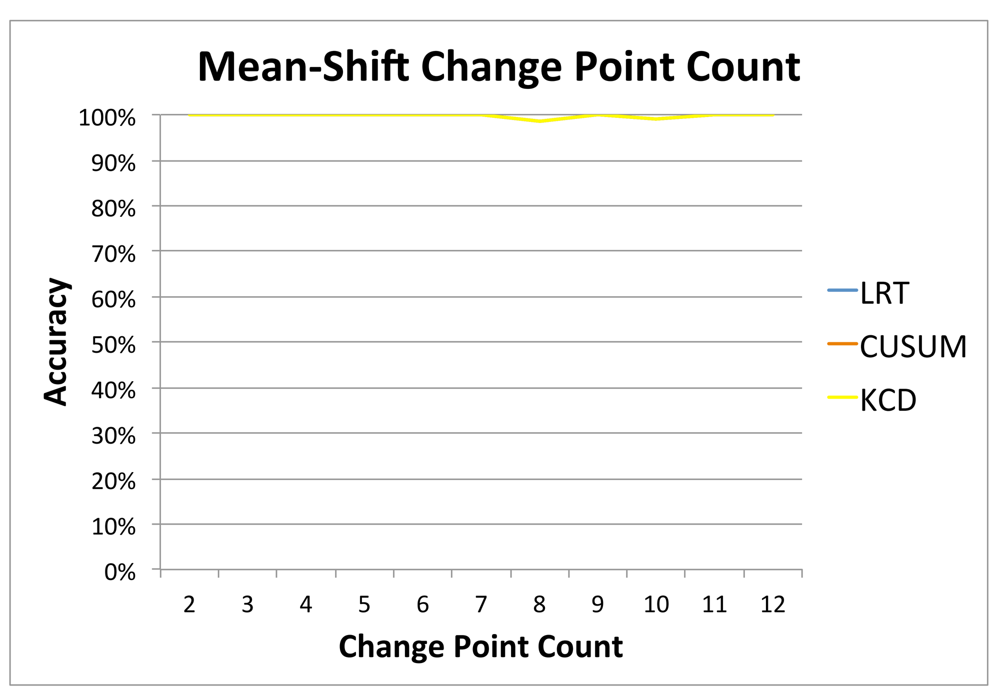


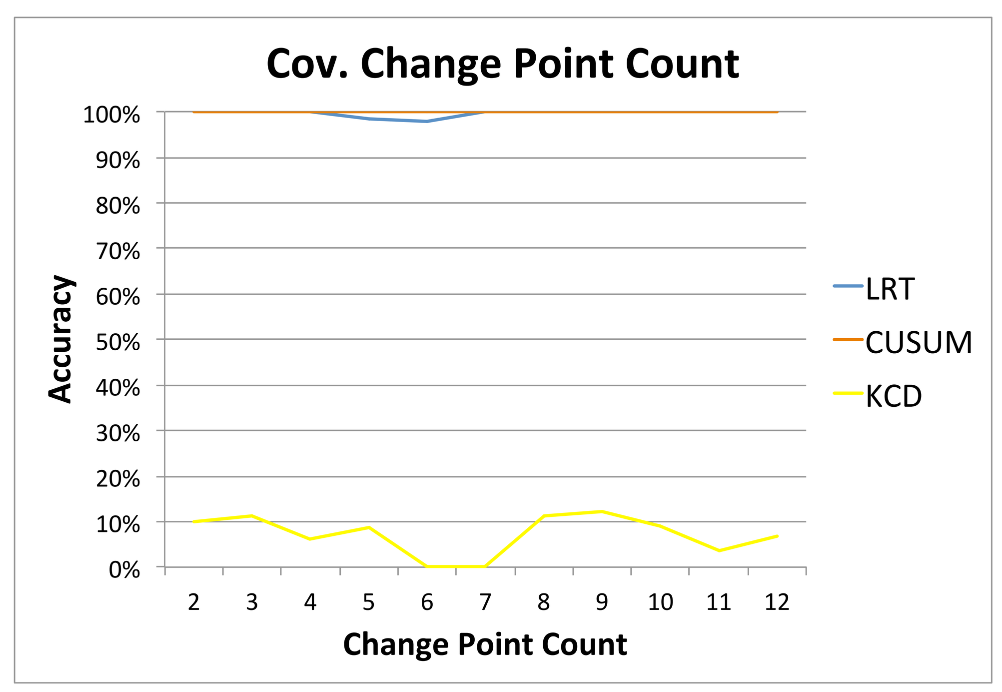


It seems the LRT and CUSUM-based algorithms are realatively insensitive to increases in dimensionality. KCD, on the other hand, seems quite sensitive with its accuracy falling to near 0% by $\boldsymbol{k}$=9.

5/7 Sensitivity to Change Point Count

Here, we simulated 3,000 bi-variate data points with 2 to 12 change points distributed evenly throughout the data set. We left the KCD window size at 400 ($\boldsymbol{m}$=200), and compared the LRT and CUSUM test statistics at the 95% confidence level.





All three algorithms seem robust to varying change points in the data. Only the covariance-based KCD implementation performs poorly with the large number of data points.

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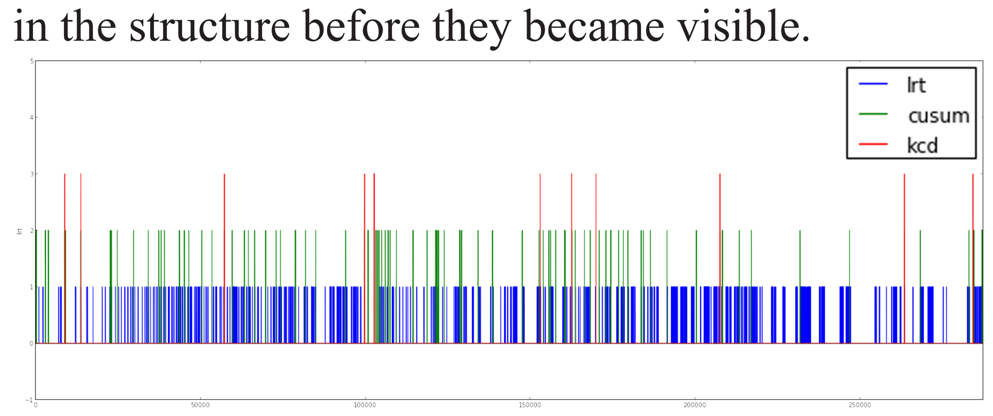
6/7 Real Applications

We applied all three algorithms to two real data sets and sought to find change points within them:

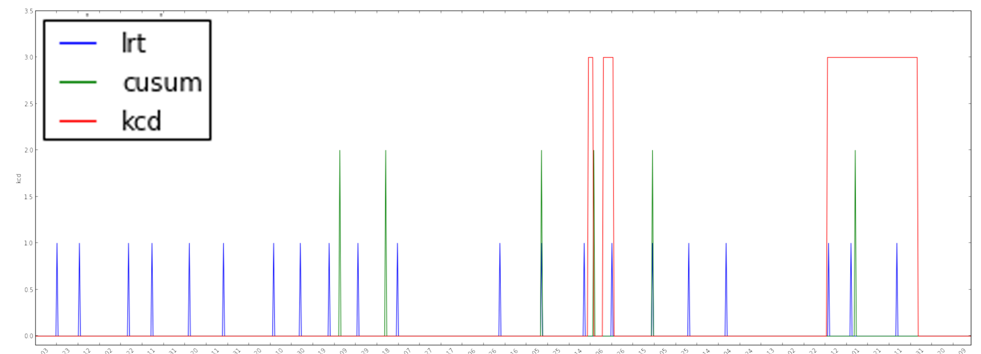
Bridge Sensor Data

Sensor data from an experiment on applying stress deformations to a bridge in a laboratory. The objective of the original data was to identify cracks in the structure before they became visible.



Mt. Gox Bitcoin Market Data

The now-defunct Mt. Gox Bitcoin exchange shared market data for Bitcoin valuations across several currencies. We analyzed two years of Bitcoin to US Dollar, Euro, GB Pound, and Polish Zloty.



7/7 Conclusions

Our performance data suggests the following results:

- The parametric LRT and CUSUM algorithms outperform the non-parametric KCD algorithm when detecting changes in covariance.
- KCD is competitive in detecting shifts in mean even with relatively small window sizes.
- LRT and CUSUM are more robust to increases in dimensionality of the data.

When applied to real data, we found the following:

- LRT detects many more change points than either CUSUM or KCD.
- KCD’s window size parameter can potentially miss change points that occur over larger periods of time.

0/0 References

[1] P. Galeano and D. Peña, “Covariance changes detection in multivariate time series,” J. Stat. Plan. Inference, vol. 137, no. 1, pp. 194–211, Jan. 2007.
[2] F. Desobry, M. Davy, and C. Doncarli, “An online kernel change detection algorithm,” Signal Process. IEEE Trans., vol. 53, no. 8, pp. 2961–2974, Aug. 2005.